Order of Operations

Without specific rules for the order in which mathematical operations like addition and multiplication are performed, it is possible to obtain two different values for an expression such as $5 + 3 \cdot 12$. For example, if the multiplication is done first, then $5 + \underbrace{3 \cdot 12} = 5 + 36 = 41$. But if the addition is done first, then $\underbrace{5 + 3} \cdot 12 = 8 \cdot 12 = 96$.

Such a situation is unacceptable in mathematics! It is resolved by providing the following standard protocol or Order of Operations Agreement.

Order of Operations Agreement

To simplify an expression with more than one operation, follow these steps:

Step 1   **<u>P</u>arentheses** – Do the operations within grouping symbols first (parentheses, brackets, braces, fraction bar, etc.), in the order given in steps 2, 3, and 4, below.

Step 2   **<u>E</u>xponents** – Do the operations indicated by exponents.

Step 3   **<u>M</u>ultiply** and **<u>D</u>ivide** – Do only multiplication and division as they occur from left to right.

Step 4   **<u>A</u>dd** and **<u>S</u>ubtract** – Do addition and subtraction as they occur from left to right.

NOTE: To help you remember the correct order, try associating the **steps** with the first letter of each word of the sentence **<u>P</u>lease <u>E</u>xcuse <u>M</u>y <u>D</u>ear <u>A</u>unt <u>S</u>ally** (the letters **PEMDAS**).

Examples:

1. $8 - 10 \div 2$ Divide first
        $8 - 5$ Subtract
          3

2. $(6 - 4)6$ Subtract within parentheses
        $(2)6$ Multiply
        12

3. $54 \div 6 \cdot 3$ Neither multiplication nor division takes precedence over the other, so do them from left to right (division first here)
        $9 \cdot 3$ Multiply
        27

Problems to Try

Directions:	Simplify	
Strategy:	Perform the operations using the Order of Operations Agreement	
	<u>Five examples</u>	<u>**You try these five**</u>

	Five examples		You try these five
1.	$7 \cdot 9 + 6 \cdot 2$ $63 + 12$ 75	Multiply first Add	1. $4 \cdot 3 + 6 \cdot 5$
2.	$25 - 6 \div 3 + 8 \cdot 4$ $25 - 2 + 32$ $23 + 32$ 55	Divide and multiply Subtract Add	2. $4 \cdot 14 - 9 \div 3 + 6 \cdot 2$
3.	$5 \cdot 9 + 9 - 6(7 + 1)$ $5 \cdot 9 + 9 - 6 \cdot 8$ $45 + 9 - 48$ $54 - 48$ 6	Add within parentheses first Multiply Add Subtract	3. $24 \div 6 + 6 - 3(5 - 3)$
4.	$3 \cdot 4^3 - 8 \cdot 3^2 + 11$ $3 \cdot 64 - 8 \cdot 9 + 11$ $192 - 72 + 11$ $120 + 11$ 131	Exponents first Multiply Subtract Add	4. $5 \cdot 2^3 - 2 \cdot 4^2 + 25 - 7 \cdot 3$
5.	$(2^2 + 2 \cdot 3)^2 + 3^2$ $(2^2 + 2 \cdot 3)^2 + 3^2$ $(4 + 2 \cdot 3)^2 + 3^2$ $(4 + 6)^2 + 3^2$ $10^2 + 3^2$ $100 + 9$ 109	First do operations <u>within parentheses</u> following proper order of operations Exponents (within parentheses) Multiply (within parentheses) Add (within parentheses) Exponents Add	5. $(3^3 - 12 \div 4)^2 + 5^2$

Answers: 1. <u>**42**</u>   2. <u>**65**</u>   3. <u>**4**</u>   4. <u>**12**</u>   5. <u>**601**</u>