Number Systems

- **Natural** (or **Counting**) Numbers $\boldsymbol{N} = \{\, 1,\, 2,\, 3, \dots \}$.

- **Whole** Numbers $\boldsymbol{W} = \{\, 0,\, 1,\, 2,\, 3, \dots \}$; they contain the natural numbers and more (namely, 0).

- **Integers** $\boldsymbol{Z} = \{\, \dots -3,\, -2,\, -1,\, 0,\, 1,\, 2,\, 3, \dots \}$; they contain the natural numbers, the whole numbers and more (namely, the negative natural numbers).

- **Rational** Numbers $\boldsymbol{Q}$ = numbers that can always be written as fractions, that is, as *ratios* of two *integers*, or equivalently, as either terminating or *repeating* decimals. Examples: $\frac{1}{2} = .5$, $-5 = \frac{-5}{1}$, $.25 = \frac{25}{100}$, $\frac{4}{9} = .44444\ldots = .\bar{4}$, $\frac{5}{12} = .41666\ldots = .41\bar{6}$ where, in the last two examples of repeating-decimal rational numbers, the "bars" over the 4 and 6 indicate that each repeats without end. They contain the natural numbers, whole numbers, integers, and more (namely, all numbers that can be expressed as fractions).

- **Irrational** Numbers $\boldsymbol{I}$ = numbers that *cannot* be written as fractions, and whose decimal representations *never repeat a pattern*, and *never terminate*. Examples: The square roots of any numbers that are not perfect squares: $\sqrt{2} = 1.414\ldots$, $\sqrt{5} = 2.236\ldots$, and special "transcendental" numbers like $\pi = 3.141\ldots$ and $e = 2.718\ldots$. $\boldsymbol{I}$ contains all real numbers that are *not rational*.

- **Real** Numbers $\boldsymbol{R}$ = any number from any of the sets $\boldsymbol{N}$, $\boldsymbol{W}$, $\boldsymbol{Z}$, $\boldsymbol{Q}$, or $\boldsymbol{I}$. The rationals, irrationals, and reals are related by the set operations $\boldsymbol{Q} \cap \boldsymbol{I} = \emptyset$, where $\emptyset$ is the empty set containing no elements, and $\boldsymbol{R} = \boldsymbol{Q} \cup \boldsymbol{I}$.

A Venn diagram illustrating the relations between these sets in given on the next page.

Real Numbers