Motion on an Inclined Plane

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Figure 1 illustrates an object of mass m at some time t, positioned on an inclined plane that makes an angle θ ($\angle ADE$ in the figure) with a horizontal line thru successive points A, D, and B. The force of gravity, equal to $m\boldsymbol{g}$ near the Earth's surface, is *vertically downward* (perpendicular to the horizontal line), acting thru the center of mass C of the block. The line CB thru the center of mass is *parallel* to the inclined plane, meeting the horizontal line at point B. Line CH thru C is *perpendicular* to the inclined plane. The horizontal line is *transverse* (a concept from elementary geometry) to the two parallel lines ED and CB, hence the angles $\angle ADE$ and $\angle ABC$ must be equal, and equal to the angle θ.

Since $\triangle BAC$ is a right triangle, it follows that $\angle ACB$ is the complement of $\angle ABC = \theta$ (the two must add up to 90°), hence $\angle ACB = 90° - \theta$, as shown in the Figure. Then, since $\triangle CHG$ is also a right triangle, $\angle HCG$ of that triangle must be complementary to $\angle ACB$, that is, $\angle HCG = 90° - (90° - \theta) = \theta$. Thus, the gravitational force (or *weight*) vector $m\boldsymbol{g}$ makes an angle θ with respect to the line CH *perpendicular* to the inclined plane. Here, $\boldsymbol{g}$ is the gravitational acceleration vector near earth's surface of magnitude $g = 9.80$ m/s^2 = 32.2 ft/s^2. This important angular relation is sometimes difficult to see in textbook diagrams, but is hopefully made clear in Figure 1.

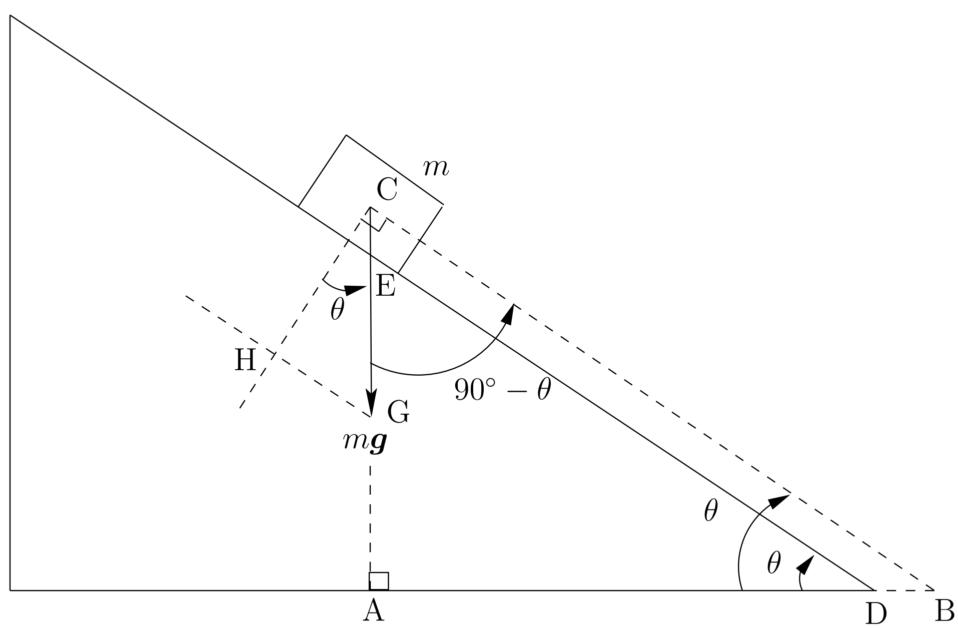
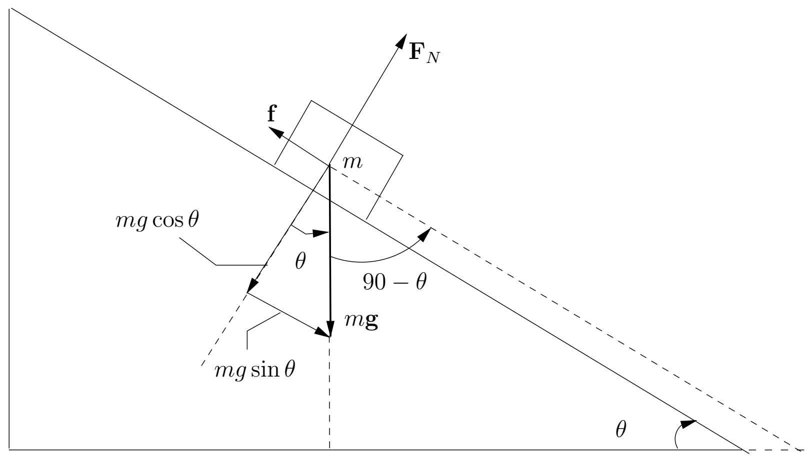


Figure 1: Angle Relations for a Mass m on Inclined Plane.

In Figure 2 on the next page, the top figure illustrates the motion of an object of mass m *down* an inclined plane with friction force $\boldsymbol{f}$ acting *upward* parallel to the incline, opposing the downward motion. In this example, the only other external forces on the block are its weight $m\boldsymbol{g}$ vertically downward, and a normal force $\boldsymbol{F}_N$ exerted by the surface of the plane upward, perpendicular to the incline.

The bottom diagram of Figure 2 illustrates motion *up* the plane, requiring an *additional* upward force $\boldsymbol{F}$ parallel to the incline to produce the motion, while the friction force $\boldsymbol{f}$, opposing this upward motion, now acts *downward* parallel to the incline.



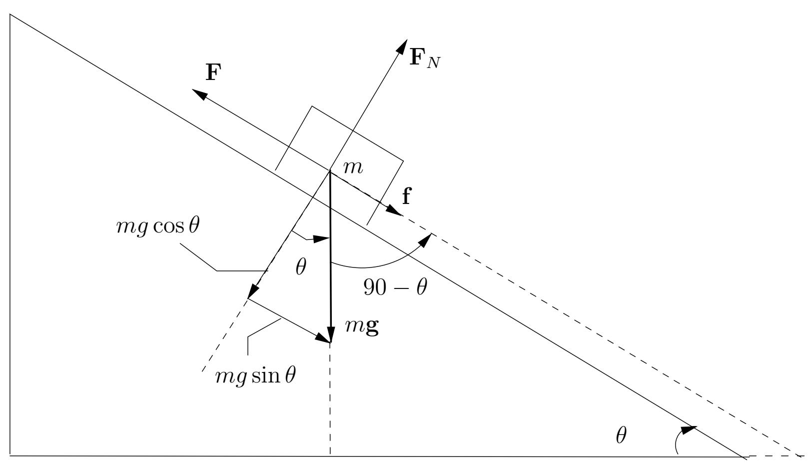


Figure 2: The friction force $\boldsymbol{f}$ may act in either direction, always *opposite* to the *direction of motion* of mass m.