ELEMENTS OF LOGIC

There are five primary logical connective symbols used in relating *statements*, or equivalently, *propositions*:

$\sim$	logical "not"	negation
$\wedge$	logical "and"	conjunction
$\vee$	logical "or"	disjunction
$\longrightarrow$	logical "if-then"	conditional
$\longleftrightarrow$	logical "if and only if"	biconditional

I. The **negation** ("**not** p", or $\sim p$) of a statement is false when the statement is true, and true when the statement is false.

II. A **conjunction** ("p **and** q", or $p \wedge q$) of two statements is *true* only when *both statements are true*, that is, $p \wedge q$ means the proposition "both a and b are true".

III. A **disjunction** ("p **or** q", or $p \vee q$) of two statements is *false* only when *both statements are false*, that is, $p \vee q$ means the proposition "at least one of the propositions a, b is true."

IV. There are four forms of "conditional" statements, each of which is *true* <u>except</u> when *a true premise implies a false conclusion*:

(a) A **conditional** proposition is stated "**if** p **then** q", written $p \longrightarrow q$.

(b) The **contrapositive** of the conditional $p \longrightarrow q$ interchanges the hypothesis and conclusion, and also negates each of them, to read $\sim q \longrightarrow \sim p$. As you will see on the last page of this note, the *conditional* and its *contrapositive* have precisely the same truth tables.

(c) The **converse** of the conditional $p \longrightarrow q$ interchanges the hypothesis and conclusion, to read $q \longrightarrow p$.

(d) The **inverse** of the conditional $p \longrightarrow q$ negates both the hypothesis and conclusion (but does not change their order), to read $\sim p \longrightarrow \sim q$. From the truth tables on the last page you will find that the *inverse* and *converse* have precisely the same truth tables.

V. The **biconditional**, stated "p **if and only if** q", and written $p \longleftrightarrow q$, is *true* when p, q are either *both true*, or *both false*.

VI. A **tautology** is a compound statement that is *true* for *every* truth value of the simple statements comprising it. The column of its truth values will contain T for every entry.

Truth tables for these relations are presented on the following pages. By definition, two pairs of logical propositions having the *same* truth tables are said to be *equivalent*. Two equivalent propositions written as a biconditional relation form a *tautology*, that is, a proposition that is *always true*. For example, you should be able to show from the truth tables on the last page that the following two compound statements are tautologies:

$$(p \longrightarrow q) \longleftrightarrow (\sim q \longrightarrow \sim p),$$

and

$$(q \longrightarrow p) \longleftrightarrow (\sim p \longrightarrow \sim q).$$

Truth Table for Statement p

p
T
F

Truth Table for **Negation** of Statement p

p	$\sim p$
T	F
F	T

Truth Table for **Conjunction** of Statements: "p **and** q"

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for **Disjunction** of Statements: "p **or** q"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table for **Biconditional** of Statements: "p **if and only if** q"

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Table for **Conditional** of Statements: "**if** p **then** q"

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for **Contrapositive** of Conditional "**if** p **then** q": "**if not** q, **then not** p"

p	q	$\sim q$	$\sim p$	$\sim q \longrightarrow \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Truth Table for **Converse** of Conditional "**if** p **then** q": "**if** q, **then** p"

p	q	$q \longrightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

Truth Table for **Inverse** of Conditional "**if** p **then** q": "**if not** p, **then not** q"

p	q	$\sim p$	$\sim q$	$\sim p \longrightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T